Hi, I'm Greg. I'm a NYC tutor! I love helping students. I tutor many subjects, assist with homework help, etc. I mainly specialize in specialized tests.

As it turns out, I haven't been able to get to do as many livestreams as I have in past years (yet, hopefully that changes). Therefore, I thought it would be fun to start a Problem Of The Day Series. I will put up a problem and leave it running for a while. You guys will then analyze it, and come up with possible solutions and alternative solutions on your own. I'll eventually post the answer in some manner.

For now we'll play it by ear how that will happen and for how long I'll leave up a problem. But right now I'm thinking of keeping the problem up maybe 2 hours minimum and maybe even in some cases 4 or 5 hours depending upon the dynamics and my situation. Unlike my AMA (Ask Me Anything) lifestream sessions, I will not be checking in every few minutes although I may from time to time join into the discussion. Again, the idea is for you guys to discuss out the problem.

Please be respectful to each other in this endeavor and let's make this fun, educational and forward-thinking. Keep the comments within the spirit of what I'm doing here. Please email me at GregsTutoringNYC@gmail.com if needed.

HERE'S THE PROBLEM: <-----
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A messenger biked from Manhattan to Brooklyn at 10 miles per hour. Upon delivery the messenger immediately returned home by the same exact route in reverse at a speed of 15 miles per hour. What was the messenger's average speed for the whole trip?

HERE'S THE SOLUTION:
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Given speed S1 and speed S2, the average speed OVER THE SAME TIME is:

$$am = \frac{S1 + S2}{2} \quad \text{Arithmetic mean computation}$$

So if I drove an hour at 10mph and an hour at 15mph, then my average speed with be 12.5mph.

However, when we're looking for the average speed OVER THE SAME DISTANCE that does not work. Instead we need to use the reciprocal of the normal average of the reciprocals of the speeds. That is to say in math:

$$\frac{2}{\frac{1}{S1} + \frac{1}{S2}} \quad \text{Harmonic mean computation}$$

This is often termed a few things, one being the "harmonic mean."

Therefore, in our example that would be:

$$\frac{2}{\frac{1}{10} + \frac{1}{15}} = \frac{2}{\frac{15}{150} + \frac{10}{150}} = \frac{2}{\frac{25}{150}} = \frac{2 \times 150}{25} = 12$$

In addition to the classic formula above for the harmonic mean there is also a variation formula:

$$\frac{2 \times S1 \times S2}{}$$

$S1 + S2$

Using our numbers we get:

$$\frac{2 \times 10 \times 15}{10 + 15} = \frac{30 \times 10}{25} = \frac{300}{25} = 12$$

Pick which you prefer, or use both to double check your computations.

Again, of importance here with these formulas is that the distance remains constant and not the time. This means we are not computing an arithmetic mean (what you know and love thus far) but a harmonic one. In short, instead of weighting for time we're weighting for distance.

Where are all these obtuse formulas coming from? Good ol' "d = r x t"!

If d = r x t then we also have:

$$r = \frac{d}{t} \quad \text{and} \quad t = \frac{d}{r}$$

Consider:

The time to go is:

$$Tg = \frac{d}{10}$$

The time to return is:

$$Tr = \frac{d}{15}$$

.: the total time, Tt, is Tg + Tr:

$$Tt = \frac{d}{10} + \frac{d}{15} = d\left(\frac{1}{10} + \frac{1}{15}\right)$$

Our average speed must consider the distance going and the distance returning, in other words the total distance is 2d since each respective distance is the same length.

Since r = d / t we're looking for the total distance over the total time. We know both of those, so let's just do it:

$$\text{Average total speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{Tt} = \frac{2d}{d\left(\frac{1}{10} + \frac{1}{15}\right)} = \frac{2}{\frac{1}{10} + \frac{1}{15}}$$

This is none other than the particular harmonic equation we pulled out of a hat and used earlier!

If it’s too conceptual approaching this as strictly as the above (no doubt the formula is looking weird), you could always plug in real numbers. Since the two speeds we’re looking at are 10 and 15, then just pick a number that is a multiple of them. In particular it is often useful that the chosen distance just be their LCM. In this case the LCM is 30. This will represent the distance involved. We don’t know the real distance but it does not matter; all that matters is that we maintain the same distance going and returning!

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As $t = d / r$ this means $Tg = 30 / 10 = 3$ and $Tr = 30 / 15 = 2$

.: $Tt = Tg + Tr = 3 + 2$
.: Total Distance $Dt = 30 + 30 = 60$

As $r = d/t$.: $60 / 5 = 12$

– Greg / GregsTutoringNYC@gmail.com LLAP ☮