

Hi, I'm Greg. I'm a NYC tutor! I love helping students. I tutor many subjects, assist with homework help, etc. I mainly specialize in specialized tests.

As it turns out, I haven't been able to get to do as many livestreams as I have in past years (yet, hopefully that changes). Therefore, I thought it would be fun to start a Problem Of The Day Series. I will put up a problem and leave it running for a while. You guys will then analyze it, and come up with possible solutions and alternative solutions on your own. I'll eventually post the answer in some manner.

For now we'll play it by ear how that will happen and for how long I'll leave up a problem. But right now I'm thinking of keeping the problem up maybe 2 hours minimum and maybe even in some cases 4 or 5 hours depending upon the dynamics and my situation. Unlike my AMA (Ask Me Anything) lifestream sessions, I will not be checking in every few minutes although I may from time to time join into the discussion. Again, the idea is for you guys to discuss out the problem.

Please be respectful to each other in this endeavor and let's make this fun, educational and forward-thinking. Keep the comments within the spirit of what I'm doing here. Please email me at GregsTutoringNYC@gmail.com if needed.

HERE'S THE PROBLEM: <—  
=====

What is the surface area to volume ratio of a sphere with a diameter that is a cube's diagonal of a cube that has a side length of  $\sqrt{3}$ ? (Note:  $\sqrt{\quad}$  means square root)

HERE'S THE SOLUTION:  
=====

Given: The diameter of a sphere is the diagonal of a cube.

If you take a face of the cube we see that it is  $\sqrt{3} \times \sqrt{3}$ . A diagonal across a face could be found by using the Pythagorean Theorem:

$$c^2 = (\sqrt{3})^2 + (\sqrt{3})^2 = 3 + 3 = 6$$
$$c = \sqrt{6}$$

You could have also calculated this considering the Pythagorean Triple 1-1- $\sqrt{2}$  which would have provided you  $\sqrt{3}-\sqrt{3}-\sqrt{3}\sqrt{2}$  which is  $\sqrt{3}-\sqrt{3}-\sqrt{6}$ .

Unfortunately this is the diagonal of the face of the cube. This is different than the diagonal of the cube itself. There are 4 diagonals to a cube. For instance, if you're facing a cube on a level flat surface one of its diagonals can be considered as a line from the top front right vertex to the bottom back left vertex. Let's call this Line D.

The formula for the diagonal of a cube is  $s\sqrt{3}$  where  $s$  is the side length of the cube.

∴ For this cube:  $s\sqrt{3} = \sqrt{3} \times \sqrt{3} = 3$  ∴ 3 is the diagonal length for this cube

∴ 3 is the diameter  $d$  of the sphere  
∴  $r = d / 2 = 3/2 = 1.5$  is the radius of the sphere

$$S_{\text{sphere}} = 4 \pi r^2$$
$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$\therefore S_{\text{sphere}}/V_{\text{sphere}} = \frac{4 \pi r^2}{\frac{4}{3} \pi r^3} = \frac{3}{r} = \frac{3}{1.5} = 2 \quad (\text{Note: } 3/r \text{ is universally true!!})$$

For those who did it longhand:

$$S_{\text{sphere}} = 4 \pi r^2 = 4 \pi 2.25 = 9 \pi$$
$$V_{\text{sphere}} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi 3.375 = 4.5 \pi$$

$$\therefore 9 \pi / 4.5 \pi = 2$$

You could have also computed the cube diagonal longhand if you didn't know the formula is  $s\sqrt{3}$ . To do so is described below.

As it turns out the cube's diagonal can be considered the hypotenuse of a right triangle, with one leg being the side edge of the cube and the other leg being the diagonal of a face of the cube. In the example of Line D given earlier, the one leg would be the diagonal across the bottom face, and the other leg would be the front rightmost edge of the front face. We already know these two values, the former would be  $\sqrt{6}$  (which we computed) and the latter  $\sqrt{3}$  (which we were given) respectively. Therefore, we can just use the Pythagorean Theorem again:

$$c^2 = (\sqrt{3})^2 + (\sqrt{6})^2 = 3 + 6 = 9 \therefore c = \sqrt{9} = 3$$

This is the same value that we computed above, therefore the rest would be the same too.

A SHSAT problem may not always be as involved as this, however, many variant questions can be asked of you using all the different aspects of math that this problem uses.

It could also be handy to memorize the additional formulas that came up in solving this problem regarding the diagonal of a cube and also regarding the surface area of a cube to its volume in an effort to save time but as can be seen they can be computed if necessary.

You can't escape that you should know the surface area and the volume of a cube though!!!

- Greg / GregsTutoringNYC@gmail.com LLAP ☺