Hi, I'm Greg. I'm a NYC tutor! I love helping students. I tutor many subjects, assist with homework help, etc. I mainly specialize in specialized tests.

As it turns out, I haven't been able to get to do as many livestreams as I have in past years (yet, hopefully that changes). Therefore, I thought it would be fun to start a Problem Of The Day Series. I will put up a problem and leave it running for a while. You guys will then analyze it, and come up with possible solutions and alternative solutions on your own. I'll eventually post the answer in some manner.

For now we'll play it by ear how that will happen and for how long I'll leave up a problem. But right now I'm thinking of keeping the problem up maybe 2 hours minimum and maybe even in some cases 4 or 5 hours depending upon the dynamics and my situation. Unlike my AMA (Ask Me Anything) lifestream sessions, I will not be checking in every few minutes although I may from time to time join into the discussion. Again, the idea is for you guys to discuss out the problem.

Please be respectful to each other in this endeavor and let's make this fun, educational and forward-thinking. Keep the comments within the spirit of what I'm doing here. Please email me at GregsTutoringNYC@gmail.com if needed.

HERE'S THE PROBLEM: <-----
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What is the range of perfect squares between 144 and 1444?

HERE'S THE SOLUTION:
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A perfect square is an integer (say 9) that is the square of an integer (3^2).

What does \|144 bring us close to or exactly to? (Note \| means square root.)

You should all know 12 x 12 = 144

But what does \|1444 bring us close to or exactly to?

Well, you should also know that 40 x 40 = 1600, therefore the root sought doesn't get as high as 40, and you know 30 x 30 = 900 which is too low even in consideration of squaring.

.: It looks like 39 wouldn't work as it still seems too high (but it might work we just don't know yet) so let's try 38 as a first try, especially as 8x8=64 (using 8 as it is the ones digit from 38) and we're looking for a 4 in the final product's ones digit (which 64 has and will give us once the multi-digit multiplication is done). Note that we also know that it is not 39 or 37 because squares of even numbers are even:

38 x 38 = 1444

Bingo, got it on the first try!

So we're looking at all the perfect square integers between 12 and 38 .: from 13 to 37.

To find the range of a bunch of numbers, order them, and then subtract the highest number from the lowest number.

.: 37 - 13 = 24

But this is not our answer. This is the answer:

.: 37^2 - 13^2 = 1369 - 169 = 1200

The above used a hunt-and-peck ad hoc binary search to find the square root. A more algorithmic approach to find the square root of a 3 or 4 digit perfect square follows.

Step 1: Consider the perfect squares from 1 through 9:

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1^2 =  1
2^2 =  4
3^2 =  9
4^2 = 16
5^2 = 25
6^2 = 36
7^2 = 49
8^2 = 64
9^2 = 81
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Note: The ones column we have 1, 4, 9, 6 and then 5 and then the same list in reverse 6, 9, 4, 1

Step 2: Consider the ones column of the number in question:

.: the last 4 in 1444

Step 3: Find which perfect squares have the same ones digit

.: 2 and 8

Note: 2+8 = 10, this will always be the case in our choices, making it easier to do this

Our answer is going to end in one of these, and we will choose between them by either picking the lower one or the higher one.

Step 4: We’re going to now ignore the 2 rightmost digits and focus on the 2 leftmost digits (or one leftmost digit in the case of a 3 digit number)

.: 14 for our example

Step 5: Look through the perfect square list above for a number just below our target

.: 4x4=16 is too high but 3x3=9 is the choice just just below 14

.: 3 is the first digit of our answer

Step 6: We need consider the two values from Step 3. That is, is the answer 32 or 28?

What we’re going to do is multiply the first digit (3) by itself plus one (3+1 = 4)

.: 3 x 4 = 12

Step 7: Compare the value from Step 6 with the value from Step 4.

If Step 4’s value is less than Step 6’s value we use the lower number from Step 3.
If Step 4’s value is higher than Step 6’s value we use the higher number from Step 3.

.: 14 > 12 so we use 8 .: the number that was squared is 38

This process appears tedious but with practice can be a fairly seamless and fast process. However it is specific to 3 or 4 digit perfect squares.

As another example, consider 5476:

* Step 2&3: Last digit 6 means our choices are 4 or 6
* Step 4&5&6: Consider 54, 8x8=64 too high, 7x7=49 .: use 7 .: 7x8=56
* 54 < 56 .: use 4 .: 74

Consider 9801:

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* Step 2&3: Last digit 1 means our choices are 1 or 9
* Step 4&5&6: Consider 98, 10x10=100 too high, 9x9=81 .: use 9 .: 9x10=90
* 98 > 90 .: use 9 .: 99

Consider 5625:

* Step 2&3: The last digit is a 5 (for this example, the complement number would have also been 5, so we’re left with a 5 for sure)
* Step 4&5&6: Consider 56, 8x8=64 too high but 7x7=49 next lowest .: use 7 .: 7x8=56
* 56 = 56, besides 5 is the only choice from Step 3 .: 75

It’s easy to do but it’s also a specific algorithm.

– Greg / GregsTutoringNYC@gmail.com   LLAP ☮